

SELF-TUNING FUZZY-PID TECHNIQUE FOR MOBILE MANIPULATOR ROBOT CONSIDERING CENTRAL MASS

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ABSTRACT

This research discusses the modeling of a 4-wheel mobile platform equipped with a mounted manipulator and proposes the self-tuning Fuzzy-PID control law for the system. A challenge with a Mobile Manipulator occurs when the manipulator performs tasks while the mobile robot is in motion, which changes the location of the center of mass (CoM) and becomes unstable in control. To address this issue, the dynamics modeling is formulated to account for the CoM displacement effect. This modeling approach estimates the effect of CoM on the motion. Based on this model, a suitable Fuzzy-PID control strategy is designed. This controller can improve the motion efficiency and compare the trajectory of the Mobile Manipulator using a conventional PID controller.

Keywords: Mobile manipulator, PID, Fuzzy-PID, displacement of CoM.

1. INTRODUCTION

In the era of Industry 4.0, manual jobs are gradually being replaced by robots. Major types of robots are being developed with high mobility, among which mobile manipulators (MM) [1] have been gaining significant attention in recent years. These robots bring substantial economic productivity and keep humans safe through applications in agriculture [2], transportation [3], etc. To perform such complex tasks, the robot must combine the flexibility of a manipulator robot with the mobility of a mobile platform. However, controlling a mobile manipulator faces several challenges due to the integration of two moving components within a system: the mobile platform and the manipulator. While proportional-integral-derivative (PID) controller is commonly used for simple models due to their simplicity and convenience, they fail to maintain stability because nonlinear effects arise from the simultaneous movement of both components. To address this issue, a self-tuning PID parameter using a fuzzy rule is proposed.

In the field of manipulator robot control, several studies have explored and compared different PID-based control methods to improve trajectory tracking performance. In [4], [5], the authors evaluate the control accuracy of three approaches: conventional PID, Fuzzy-PID, and Fuzzy-PD+I through simulation on a 6-DOF manipulator. The Fuzzy PD+I structure proposed in this work is functionally similar to the fuzzification process of a conventional Fuzzy controller, where the proportional and derivative gains are used as inputs to the Fuzzy system, and the output is added to the integral gain. Notably, this approach does not employ fuzzy rules for online tuning of PID parameters, distinguishing it from adaptive Fuzzy-PID methods. Meanwhile, studies [6], [7] introduce and analyze the performance of PID, Fuzzy-PID, and Orthogonal Fuzzy-PID controllers. These works use angular error and its derivative as inputs to the Fuzzy system, which represents a variation from conventional Fuzzy-PID structures typically based on positional error. The orthogonal Fuzzy structure is designed to improve controller flexibility and performance in dynamic environments.

Several studies have applied Fuzzy-based PID controllers to improve the performance of mobile robot (MR) motion and trajectory tracking. In [8], a Fuzzy-PID controller is proposed for obstacle

avoidance and position control of a mobile robot under varying dynamic conditions. Similarly, the study in [9] implements a Fuzzy-PI controller for robot navigation and tracking. The controller uses error and the derivative of error as inputs, and the outputs are $\Delta K_p, \Delta K_i$. The comparison between the conventional PI and the Fuzzy-PI shows that the Fuzzy-based approach achieves more effective tracking. However, this study does not include a saturation block to limit the control output, which may lead to integrator windup, potentially rendering the real system incapable of achieving ideal control. To address this limitation, the use of an anti-windup mechanism is recommended. In study [10], the author discusses the design of the Fuzzy-PID controller, which aims to reduce rise time, eliminate steady-state error quickly, and avoid overshoot during robot motion. Furthermore, [11] presents a Fuzzy-PID controller for trajectory tracking of a differential drive mobile robot along a circular path with a radius of 1 meter. The simulation results highlight the effectiveness of the Fuzzy-PID compared to a conventional PID controller in maintaining accurate trajectory tracking. This research [12] depicts the controller for a mobile manipulator with passive suspension. Its purpose is to reduce the oscillation error of the system through no-feedback control, PID control and Fuzzy-PID control.

In this paper, the problem of mobile manipulators is the center of mass (CoM) location changing, and the friction forces of each wheel change. Furthermore, the distance between CoM and the instantaneous center of rotation (ICR) is a non-zero parameter, so the system is more nonlinear than a mobile robot system. Considering the above related works and the manipulator problem, the present study addresses contributions including: modeling manipulator motion of the equation, modeling mobile platform with CoM location changing, designing Fuzzy-PID, and simulation the path of the system to compare the effective performance of conventional PID and Fuzzy-PID.

This paper is organized as follows: Section 2 considers the model of the mobile manipulator, including kinematics and dynamics. In section 3, the Fuzzy-PID controller is proposed for the mobile platform's angular and linear velocity, and Fuzzy-PID for the manipulator's joints. Section 4 shows the charts comparing the effectiveness between conventional PID and Fuzzy-PID. In the final section, the conclusion and discussion are proposed in Section 5.

2. MODELING

2.1. Manipulator modeling

2.1.1. Compute the location of the Center of Mass

The position of each link's center of mass in the global frame is obtained via homogeneous transformation matrices, which is as same as [13], [14]. Define the following transformation matrices $T_i^0 \in \mathbb{R}^{4 \times 4}$:

$$T_i^0 = T_1^0 T_2^1 \dots T_i^{i-1} = \begin{bmatrix} R_i^0 & d_i^0 \\ 0^T & 1 \end{bmatrix} \quad (1)$$

where $R_i^0 \in \mathbb{R}^{3 \times 3}$ denotes the rotation matrix and $d_i^0 = [0 \ 0 \ l_i]^T \in \mathbb{R}^3$ indicates the displacement between the base and the CoM of i^{th} frame. Using Equation (1), the positions $P_{mC_i}^0 = [x_{mC_i} \ y_{mC_i} \ z_{mC_i}]^T$ of the centers of mass of the manipulator's links are:

$$P_{mC_i}^0 = T_i^0 \begin{bmatrix} r_{C_i} \\ 1 \end{bmatrix} + [d_x \ 0 \ h \ 1]^T \quad (2)$$

with r_{C_i} is the local location of the CoM of the link i^{th} ; d_x and h are the displacement from the coordinate of the mobile platform to the manipulator's following x and z , in Figure 1.

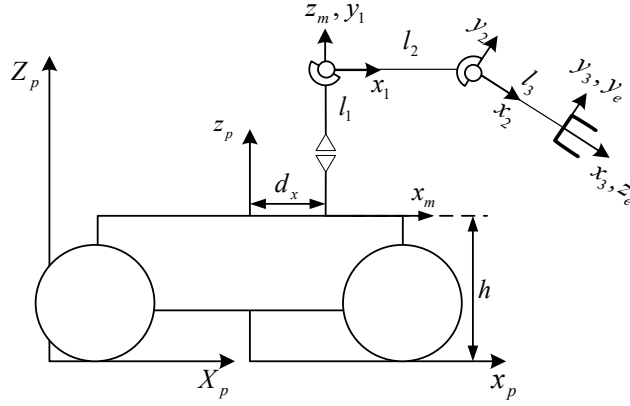


Figure 1. Coordinate of the mobile manipulator robot in the XZ plane

The center of mass location of the mobile manipulator $p_{CoM} = [x_{CoM} \ y_{CoM} \ z_{CoM}]^T$ can be expressed as:

$$p_{CoM} = \frac{\sum_{i=1}^3 m_{m_i} p_{mC_i}^0 + m_p p_{pC}}{\sum_{i=1}^3 m_{m_i} + m_p} \quad (3)$$

with m_{m_i} is the mass of the manipulator's i_{th} link; m_p is the mass of the platform and $p_{pC} \in \mathbb{R}^3$ is the center of mass location of the platform. In this case, the platform is assumed to be symmetrically designed, so its center of mass is located at the geometric center, which leads to the $p_{pC} = [0 \ 0 \ h/2 \ 1]^T$. Because the CoM position of the platform is at the origin of its x and y axes, the center of mass of the whole system is dependent on the manipulator's.

2.1.2. Manipulator's dynamic modeling

In this section, the Lagrange-Euler method in [14], [15], [16] can be used to describe the motion of equation of n-DOF manipulator.

$$M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m) + G_m(q_m) = \tau_m \quad (4)$$

with $M_m(q_m) \in \mathbb{R}^{3 \times 3}$ is the inertia matrix, $C_m(q_m, \dot{q}_m) \in \mathbb{R}^3$ is the centripetal and Coriolis matrix, $G_m(q_m) \in \mathbb{R}^3$ is the gravity matrix; q_m is the joint variable matrix, and τ_m denotes n torques of the manipulator's joints. In this study, the 3-DOF articulated manipulator is considered, so that those matrices can be shown as follows [15]:

$$M_m(q_m) = \begin{bmatrix} M_{m(1,1)} & 0 & 0 \\ 0 & M_{m(2,2)} & M_{m(2,3)} \\ 0 & M_{m(3,2)} & M_{m(3,3)} \end{bmatrix}; C_m(q_m) = \begin{bmatrix} C_{m1} \\ C_{m2} \\ C_{m3} \end{bmatrix}; G_m(q_m) = \begin{bmatrix} 0 \\ G_{m2} \\ G_{m3} \end{bmatrix} \quad (5)$$

2.2. Mobile platform modeling

2.2.1. Kinematics model

The mobile platform in this study is a skid-steering wheel mobile robot with non-holonomic constraints. The robot has 3-DOF, but it only has two input variables, so its kinematics model is based on researches [17], [18], [19], [20]. Its special features include the Instantaneous Center of Rotation (ICR) and resistance forces and torques. The kinematics can be expressed as follows:

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix} \quad (6)$$

$$\eta = \begin{bmatrix} v_x \\ \omega_z \end{bmatrix} = r \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{W} & \frac{1}{W} \end{bmatrix} \begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix} \quad (7)$$

where $\omega_L = \omega_1 = \omega_2$, $\omega_R = \omega_3 = \omega_4$; ω_i with $i=1,2$ indicate the front and rear wheels on the left side and $i=3,4$ are right side, which are shown in Figure 2. The skid-steering mobile robot has two linear input variables. v_x and angular ω_z velocities, meanwhile the Equation (6) needs three. To complete the kinematics model, the nonholonomic constraint and its Pfaffian form are proposed as follows:

$$v_y + x_{ICR} \omega = 0 \quad (8)$$

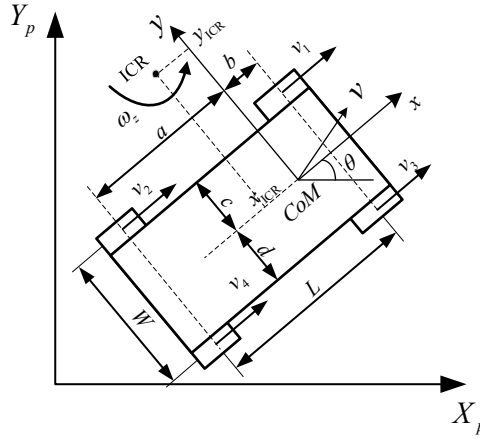


Figure 2. Coordinate of the mobile manipulator robot in the XY plane

$$\begin{bmatrix} -\sin \theta & \cos \theta & x_{ICR} \end{bmatrix} \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \end{bmatrix} = A(q) \dot{q} = 0 \quad (9)$$

Substitute Equation (8) into Equation (6), and the kinematics matrix can be expressed as:

$$\dot{q}_p = S(q_p) \eta = \begin{bmatrix} \cos \theta & x_{ICR} \sin \theta \\ \sin \theta & -x_{ICR} \cos \theta \\ 0 & x_{ICR} \end{bmatrix} \begin{bmatrix} v_x \\ \omega_z \end{bmatrix} \quad (10)$$

where $\dot{q}_p = [\dot{X} \ \dot{Y} \ \dot{\theta}]^T$, and x_{ICR} denotes the distance between the coordinates of CoM and ICR along its x-axis. The x_{ICR} is considered in the CoM's coordinates, but the x_{CoM} is considered in the mobile robot's geometric center coordinate, so the $x_{ICR} = -x_{CoM}$.

2.2.2. Dynamics model

The platform's equation of motion can be described by the Lagrange-Euler equation, which is the same as [20] and [21]. But, in this study, the dynamic CoM caused by the manipulator's motion leads the x_{ICR} and resistance to non-constant variables.

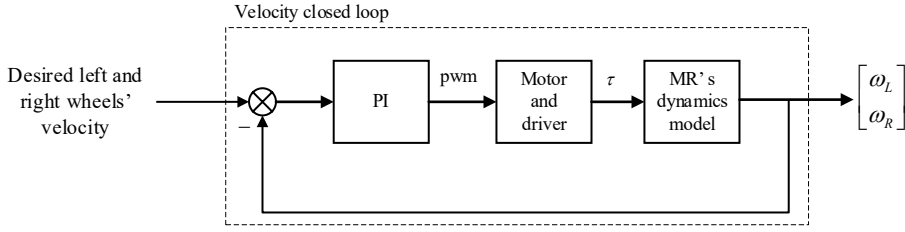


Figure 3. Coordinate of the mobile manipulator robot

$$\frac{d}{dt} \frac{\partial L_p}{\partial \dot{q}_p} - \frac{\partial L_p}{\partial q_p} = F + A^T(q_m)\lambda - R \quad (11)$$

$$\bar{M}\dot{\eta} + \bar{C}\eta + \bar{R} = \bar{E}\tau_p \quad (12)$$

where $\bar{M}$ represents the inertia matrix, $\bar{C}$ is centrifugal force, $\bar{R}$ is the resistance matrix and $\bar{E}$ denotes the input force matrix, which converts the motors' torque τ_p to total force and moment.

$$\begin{aligned} \bar{M} &= \begin{bmatrix} m_p & 0 \\ 0 & m_p x_{ICR}^2 + I \end{bmatrix}; & \bar{C} &= \begin{bmatrix} 0 & m_p \omega_z x_{ICR} \\ -m_p \omega_z x_{ICR} & 0 \end{bmatrix} \\ \bar{R} &= \begin{bmatrix} F_{Rx}(q_p) \\ M_{Rz}(q_p) - x_{ICR} F_{Ry}(q_p) \end{bmatrix}; & \bar{E} &= \frac{1}{r} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -\frac{W}{2} & -\frac{W}{2} & \frac{W}{2} & \frac{W}{2} \end{bmatrix} \\ \tau_p &= [\tau_1 \quad \tau_2 \quad \tau_3 \quad \tau_4]^T. \end{aligned} \quad (13)$$

Equation (13) has components of $\bar{R}$, which can be expressed as follows:

$$\begin{aligned} F_{Rx} &= \sum_{i=1}^4 F_{iRx}(v_{ix}) = \sum_{i=1}^4 \mu_{ix} N_i(v_{ix}) \\ F_{Ry} &= \sum_{i=1}^4 F_{iRy}(v_{iy}) = \sum_{i=1}^4 \mu_{iy} N_i(v_{iy}) \\ M_{Rz} &= b \sum_{i=1,3} F_{iRy}(v_{iy}) - a \sum_{i=2,4} F_{iRy}(v_{iy}) + d \sum_{i=3,4} F_{iRx}(v_{ix}) - c \sum_{i=1,2} F_{iRx}(v_{ix}) \end{aligned} \quad (14)$$

where $a = L/2 + x_{CoM}$; $b = L/2 - x_{CoM}$; $c = W/2 - y_{CoM}$; $d = W/2 + y_{CoM}$.

The problem appears when the CoM position is different from the geometric center of the mobile robot. The reaction forces cannot be expressed in as same way as [20] and [21], so these need to be calculated using the method from [22]. This study considers the rectangular table with the load not located at the center point. The wheels' reaction force can be indicated as:

$$\begin{aligned} N_1 &= mg \frac{d \times a}{WL}; & N_2 &= mg \frac{d \times b}{WL} \\ N_3 &= mg \frac{c \times a}{WL}; & N_4 &= mg \frac{c \times b}{WL} \end{aligned} \quad (15)$$

where the ensuring non-overturning condition is at $N_1, N_2, N_3, N_4 \geq 0$

2.2.3. Motor and driver

This section considers the block between the controllers and the dynamics model, which can be found by hardware experiments. Refer in Figure 4 and 5, the motor and driver block include a power circuit based on the BTS7960, supplies current to the DC motor according to pwm control signal. The block diagram of DC motor's transfer function.

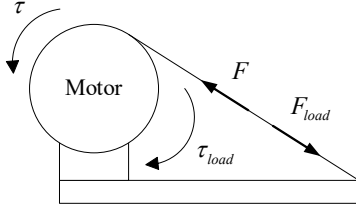


Figure 4. Schematic diagram of the experimental model

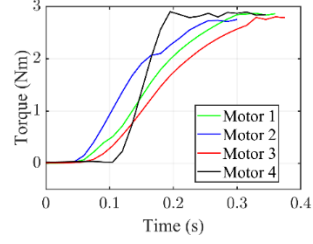


Figure 5. Motors' torque response experiment

The purpose of this experience is to determine the relationship between PWM input and torque output. In Figure 4 and 5, when a $PWM = 100$ is applied, the force gradually increases until it reaches a steady state at approximately $2.8Nm$. In this point, $\tau = \tau_{load}$, and $\omega = 0$.

The resulting transfer function of this experiment includes both BTS7960 motor driver and DC transfer function can be expressed as in Figure 5, which reach at approximately $2.8Nm$, but they have different response time and hard to control.

3. DESIGN OF CONTROLLER

3.1. PID controller

A proportional (P)-integral (I)-derivative (D) controller [23], [24] is a common controller and is used widely by a simple and easily implemented structure. In contrast, a PID controller is hard to stabilize a nonlinear system such as a mobile robot. In the mobile robot model, the kinematics Equation (10) has two inputs and three outputs, so that it can be used as inverse kinematics to transform global error to local error. It has an unmeasured variable x_{ICR} which cannot be observed by any sensors, so in the controller, the x_{ICR} can be assumed that zero, the path tracking controller based on conventional PID of the mobile platform, can be expressed as the inverse of Equation (10):

$$\begin{bmatrix} v_x \\ \omega_z \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} K_{pX}e_x + K_{d\theta}\dot{e}_x \\ K_{pY}e_y + K_{d\theta}\dot{e}_y \\ K_{p\theta}e_\theta + K_{d\theta}\dot{e}_\theta \end{bmatrix} \quad (16)$$

The mobile platform uses a Fuzzy-PD controller for the position controller, and it is hard to use integral gain, so the conventional PI controller is used in the velocity loop, in Figure 3, but the manipulator uses the PID controller in this paper.

3.2. PID controller Fuzzy-PID controller

The mobile manipulator system has a large number of nonlinear components, especially the CoM changing in this study, which makes it more complex and harder to implement conventional PID for the tracking problem. To improve it, the Fuzzy-PID controller with a simple structure is proposed.

In this approach, error and derivative error are used as input, and $\Delta K_p, \Delta K_i, \Delta K_d$ are considered as outputs. The Fuzzy-PID K_p, K_i, K_d parameters can be calculated as follows:

$$\begin{aligned} K_p &= K_p^0 + k_p \Delta K_p \\ K_i &= K_i^0 + k_i \Delta K_i \\ K_d &= K_d^0 + k_d \Delta K_d \end{aligned} \quad (17)$$

where K_p^0, K_i^0, K_d^0 are the initial PID parameters and k_p, k_i, k_d are amplified coefficients.

The inputs and outputs of the platform's fuzzy rules share the same range $[-1; 1]$. Its corresponding

membership functions are indicated in Figures 7 and 8. However, for the manipulator, the input ranges are defined as $[-0.5; 0.5]$ for error e and $[-0.1; 0.1]$ for derivative error de .

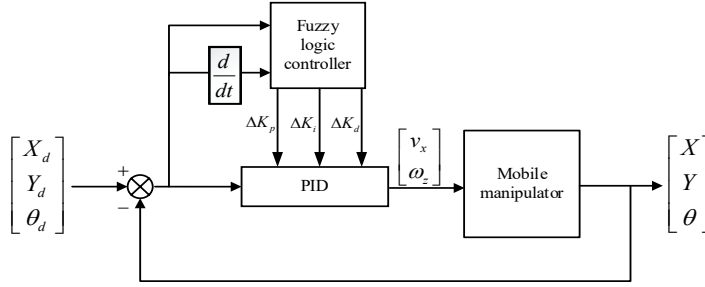


Figure 6. Block diagram of a FPID

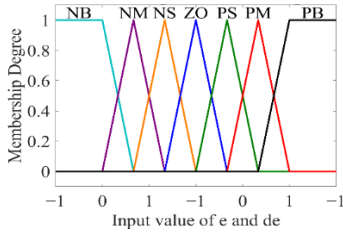


Figure 7. Input membership functions for error (e) and change of error (de)

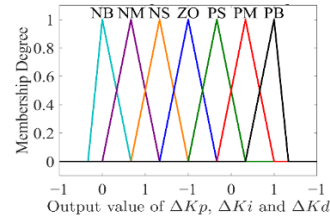


Figure 8. Output membership function for ΔK_p , ΔK_i and ΔK_d

The Fuzzy has 7×7 rules that are determined through a trial-and-error approach, which can be used for both the mobile robot and the manipulator, are presented in Tables 1, 2, and 3.

Table 1. Fuzzy rule of ΔK_p

ΔK_p		de						
		NB	NM	NS	ZO	PS	PM	PB
e	NB	NB	NB	NM	PB	PB	PS	ZO
	NM	NB	NM	NS	PM	PM	ZO	PS
	NS	NM	NS	NS	PS	ZO	PS	PS
	ZO	PM	PS	ZO	ZO	ZO	PS	PM
	PS	PS	PS	ZO	PS	NS	NS	NM
	PM	PS	ZO	PM	PM	NS	NM	NB
	PB	ZO	PS	PB	PB	NM	NB	NB

Table 2. Fuzzy rule of ΔK_i

ΔK_i		de						
		NB	NM	NS	ZO	PS	PM	PB
e	NB	NB	NB	NM	PB	PB	PS	ZO
	NM	NB	NM	NS	PM	PM	ZO	PS
	NS	NM	NS	NS	PS	ZO	PS	PS
	ZO	PM	PS	ZO	ZO	ZO	PS	PM
	PS	PS	PS	ZO	PS	NS	NS	NM
	PM	PS	ZO	PM	PM	NS	NM	NB
	PB	ZO	PS	PB	PB	NM	NB	NB

Table 3. Fuzzy rule of ΔK_d

ΔK_d		de						
		NB	NM	NS	ZO	PS	PM	PB
e	NB	NB	NB	NM	NB	ZO	PS	PM
	NM	NB	NM	NS	NM	NM	ZO	PB
	NS	NM	NS	NS	NS	NB	NM	PB
	ZO	PB	PB	PM	ZO	PM	PB	PB
	PS	PB	NM	NB	NS	NS	NS	NM
	PM	PB	ZO	NM	NM	NS	NM	NB
	PB	PM	PS	ZO	NB	NM	NB	NB

4. SIMULATION AND ANALYSIS

4.1. Simulation parameters

Table 4. Simulation parameters

arameters	Symbol	Value
Distance from MR's center to manipulator's coordinate (m)	d_x	0.4
Platform's height (m)	h	0.2
Platform's length (m)	L	0.94
Platform's width (m)	W	0.55
Platform's mass (kg)	m_p	5
Platform's inertia ($kg.m^2$)	I	5
Wheels' radius (m)	r	0.2
Max linear velocity (m/s)	v_x	2
Max angular velocity (m/s)	ω_z	6.67
Rolling resistance coefficient	μ_x	0.08
Lateral resistance coefficient	μ_y	0.55
Max wheel's torque (Nm)	τ_p	2.8
Links' length of manipulator (m)	$l_1; l_2; l_3$	0.2; 1.25; 0.8
Links' CoM position of manipulator (m)	$r_{C_1}; r_{C_2}; r_{C_3}$	0.1; 0.9375; 0.6
Links' mass of manipulator (kg)	$m_{m_1}; m_{m_2}; m_{m_3}$	0.5; 1.4; 0.6
Joints' torque (Nm)	$\tau_{m_1}; \tau_{m_2}; \tau_{m_3}$	65; 180; 40

Table 5. The controllers' parameters

Parameters	Symbol	Value
MR's initial PID parameters of the X, Y axes and θ_z	$\begin{bmatrix} K_{p(X,Y,\theta)}^0 \\ K_{i(X,Y,\theta)}^0 \\ K_{d(X,Y,\theta)}^0 \end{bmatrix}$	$\begin{bmatrix} 10 \\ 0 \\ 10 \end{bmatrix}$
MR's amplified coefficients of the X, Y axes, and θ_z	$\begin{bmatrix} k_{pX} \\ k_{iX} \\ k_{dX} \end{bmatrix}$	$\begin{bmatrix} 10 \\ 0 \\ 10 \end{bmatrix}$
The PI controller in Figure 3	$\begin{bmatrix} k_{p\omega} \\ k_{i\omega} \\ k_{d\omega} \end{bmatrix}$	$\begin{bmatrix} 10 \\ 70 \\ 0 \end{bmatrix}$
Joints' initial PID parameters	$\begin{bmatrix} K_{p\theta_{1,2,3}}^0 \\ K_{i\theta_{1,2,3}}^0 \\ K_{d\theta_{1,2,3}}^0 \end{bmatrix}$	$\begin{bmatrix} 70 \\ 50 \\ 20 \end{bmatrix}$
Joint's amplified coefficients	$\begin{bmatrix} k_{p\theta_{1,2,3}} \\ k_{i\theta_{1,2,3}} \\ k_{d\theta_{1,2,3}} \end{bmatrix}$	$\begin{bmatrix} 10 \\ 20 \\ 20 \end{bmatrix}$
Simulation sample time (ms)		10

In this section, two simulation cases are considered to verify the accurate behavior of the proposed controllers, which can be compared with the motion with and without CoM changing, and the system with Fuzzy-PID and conventional PID. The Table 4 and Table 5 indicate the mobile manipulator simulation and controller parameters.

4.2. Manipulator simulation

Figure 9 shows the motion of the manipulator and its pose, which is caused by three joints over time. They initially stand at $[0; \pi/2; -\pi/2]$ and begin going down to $[0; 0; 0]$ at 5 seconds. For a while, it rotates in the backward direction $[0; \pi; 0]$ at 40 seconds. As shown in Figures 10, 11 and 12, the Fuzzy-PID controller is applied to control the joint angle position and compared with conventional PID, including response and error. In the error picture parts, two controllers also have peaks when the reference signals move instantly, but the Fuzzy-PID has lower lines and responses faster than conventional PID.

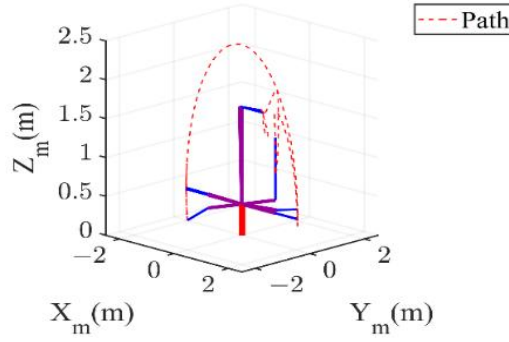


Figure 9. The manipulator's pose

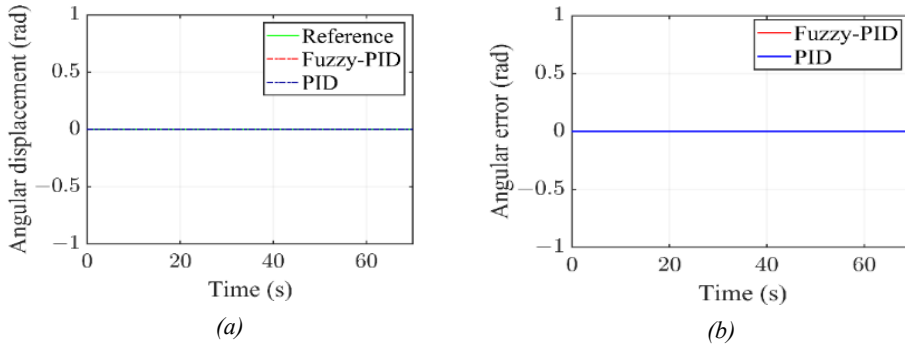


Figure 10. The (a) displacement and (b) error of the first joint's angle

Generally, in Figures 11 and 12, considering the second and third joints of the manipulator, peaks occur when the reference angles change, which causes overshoot, and then it closely reaches the equilibrium point. In detail, the Fuzzy-PID's plot responses become more stable than the PID's ones. The third joint only manipulates once at 5 seconds, but it also fluctuates, coinciding with the second joint's motion due to the dynamics modeling discussed in Equation (5).

4.3. Mobile platform simulation

In this section, motion without and with the displacement of CoM is considered. Figure 13(a) indicates the zero CoM, which only simulates the simple mobile robot. In Figure 13(b), those lines are caused by the manipulator's displacement above Figure 9, and they simulate the mobile platform's motion of the mobile manipulator.

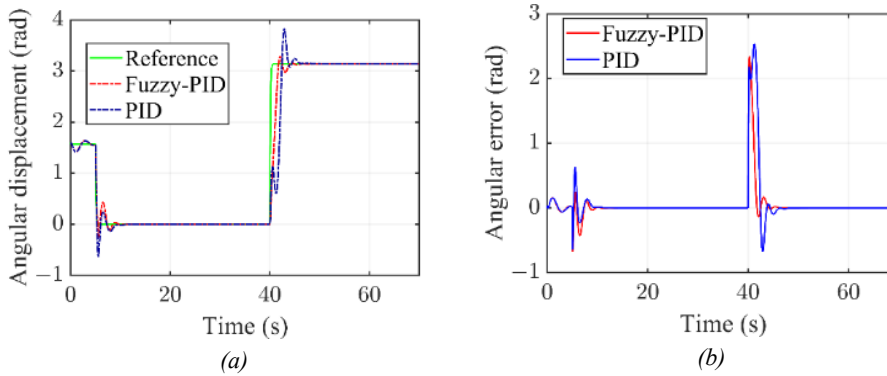


Figure 11. The (a) displacement and (b) error of the second joint's angle

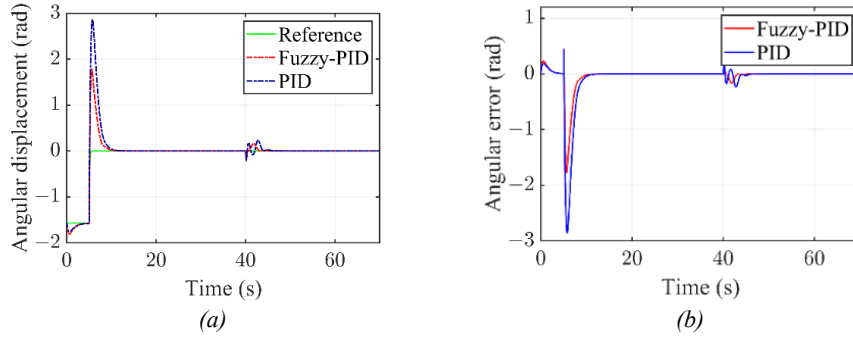


Figure 12. The (a) displacement and (b) error of the third joint's angle

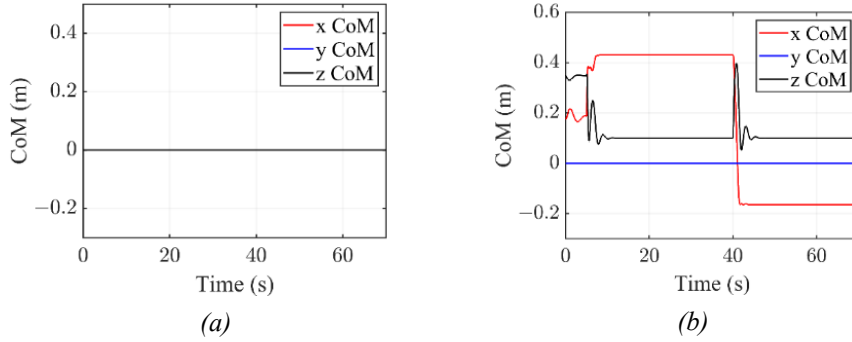


Figure 13. CoM position is considered in (a) the original of the platform case and (b) the change by the manipulator's motion case

In Figure 14, generally, the system with Fuzzy-PID has a faster response and is more stable than conventional PID, so it does not go out of the curve. In Figure 14(b), the system with PID suddenly lost control during operation, and its trajectory slowly approaches the reference path, meanwhile the Fuzzy-PID remains stable. Considering the zero-CoM case in Figure 15(a), 16(a) and 17(a), overall, the path of Fuzzy-PID has more accurate trajectory tracking than the PID's. It has the same slope as the PID in the initial stage, but when it gets close to the reference path, its slope decreases greatly and becomes more stable than the PID's.

In the CoM displacement case, to evaluate control stability and tracking performance, refer to Figures 15(b), 16(b) and 17(b), a discontinuous curve of Fuzzy-PID occurred at approximately 5 seconds, which coincides with the CoM displacement in Figure 13. Meanwhile, the trajectory under conventional PID remains continuous, so its tracking curve slope is greater than Fuzzy-PID's, but it leads to a significant deviation and loss of control from the reference path, especially during the initial response stage.

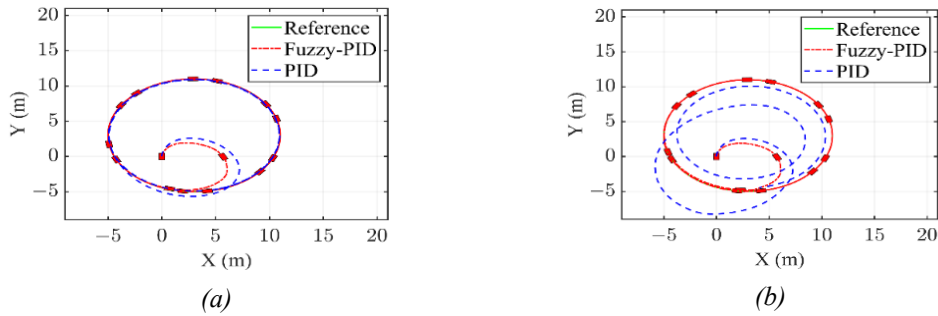


Figure 14. Tracking path applying Fuzzy-PID and PID under (a) zero CoM and (b) displacement CoM condition

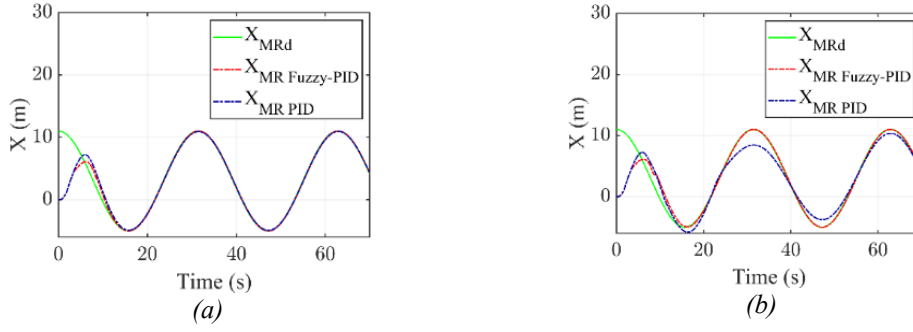


Figure 15. MR's trajectory in the X-axis using Fuzzy-PID and PID under (a) zero CoM and (b) displacement CoM condition

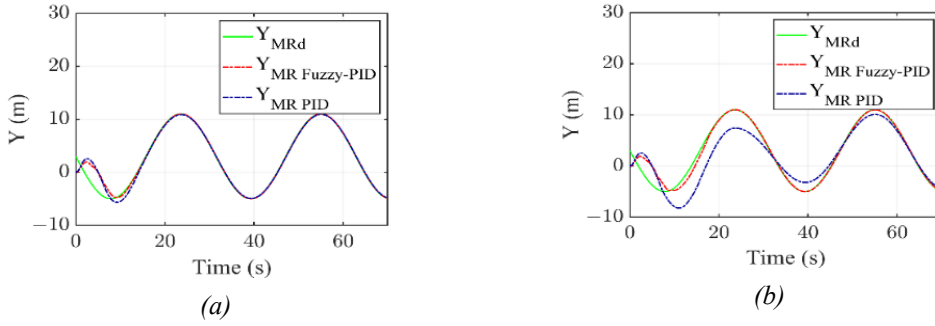


Figure 16. MR's trajectory in the Y-axis using Fuzzy-PID and PID under (a) zero CoM and (b) displacement CoM condition

Table 8 indicates a comparative analysis of the Fuzzy-PID and conventional PID under two condition: Zero CoM and Displacement CoM. The performance metrics used for comparison are steady-state error and settling time. Overall, the Fuzzy-PID demonstrates lower steady-state error and shorter settling time compared to the conventional PID. In the other hand, under displacement CoM condition, those also increase except Figure 17, especially PID's, the steady-state error reaches nearly three times higher than PID of normal condition. Furthermore, their settling time increase significantly which is almost close to the end of the simulation period, indicating a loss of control.

Discussing the error simulation after 40 seconds when the manipulator changes the CoM's pose to the backward, in Figures 18(b) and 19(b), the oscillation amplitude decreases gradually, so the deviation tracking plot of the PID controller in Figures 15(b) and 16(b) decreases too.

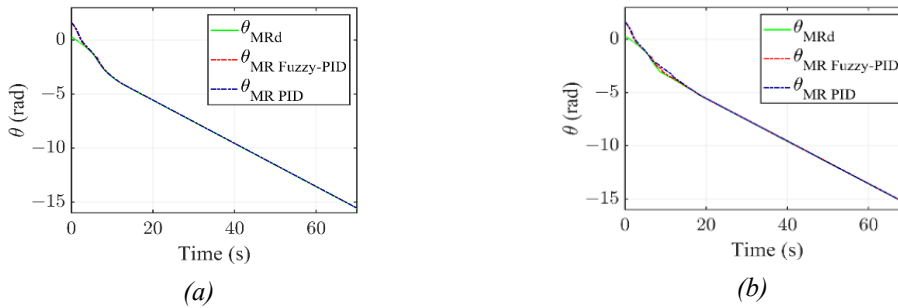


Figure 17. MR's angular using Fuzzy-PID and PID under (a) zero CoM and (b) displacement CoM condition

Table 6. Response analysis of the mobile platform

Figure	Controller	Zero CoM	Displacement CoM
Figure 15	Fuzzy-PID	Steady-state error: 0.98% Settling Time: 11.06 s	Steady-state error: 0.98% Settling Time: 14.47 s
	PID	Steady-state error: 0.7% Settling Time: 12.58 s	Steady-state error: 2.73% Settling Time: 64.94 s
Figure 16	Fuzzy-PID	Steady-state error: 0.72% Settling Time: 8.22 s	Steady-state error: 0.72% Settling Time: 17.31s
	PID	Steady-state error: 1.12% Settling Time: 13.89 s	Steady-state error: 3.76% Settling Time: 59.08s
Figure 17	Fuzzy-PID	Steady-state error: 0.13% Settling Time: 1.53 s	Steady-state error: 0.12% Settling Time: 1.5 s
	PID	Steady-state error: 0.14% Settling Time: 1.71 s	Steady-state error: 0.13% Settling Time: 1.67 s

Figure 20 shows the information about angular error. In Figure 20(a), the error under PID and Fuzzy-PID controller increase simultaneously to zero, but the Fuzzy-PID has the little sharp point at approximately second 5. In Figure 20(b), it is bigger, but, in this case, the PID's occurs larger than the Fuzzy-PID's.

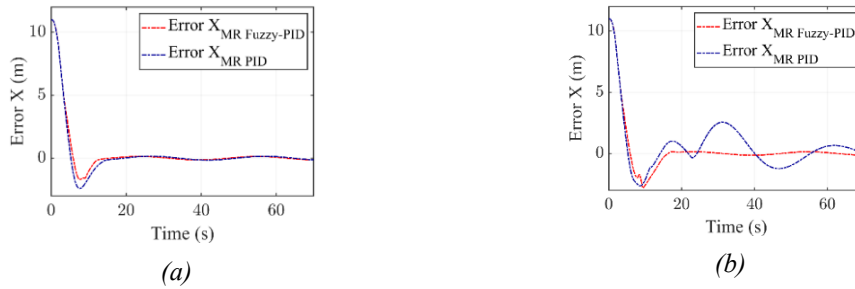


Figure 18. Error in X-axis using Fuzzy-PID and PID under (a) zero CoM and (b) displacement CoM condition

Generally, in Figure 21, before the 20th second, considering the zero CoM case, the linear velocity of the Fuzzy-PID is lower than the PID. This occurred when the platform is in the initial response and has the highest position error stage, so the velocity of PID reaches the highest and remains for a while. Meanwhile, in this period, the Fuzzy-PIDs decrease instantly and fluctuate, then they remain stable velocity faster than the PIDs. In the CoM displacement case, this period is longer and occurs more fluctuated, so the linear velocity of PID decreases dramatically, then increases slowly to the stable velocity. In contrast, the Fuzzy-PID reaches the stable velocity even faster than the PID. In Figure 22, the angular velocity plot of Fuzzy-PID has more fluctuating points than PID's because it gives priority to angle adjustment over speed response.

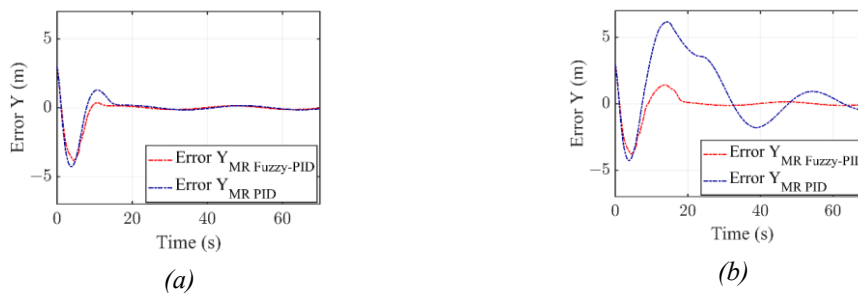


Figure 19. Error in Y-axis using Fuzzy-PID and PID under (a) zero CoM and (b) displacement CoM condition

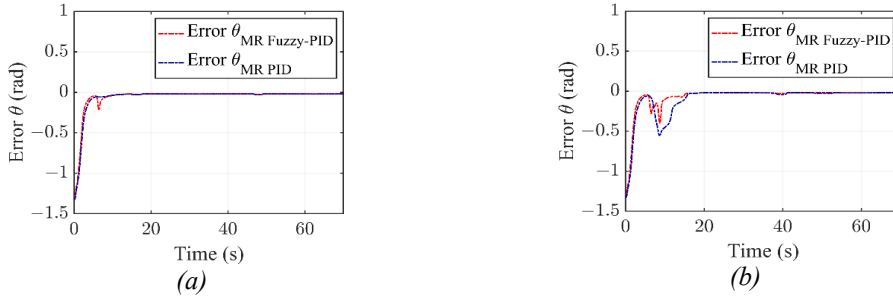


Figure 20. Angular error using Fuzzy-PID and PID under (a) zero CoM and (b) displacement CoM condition

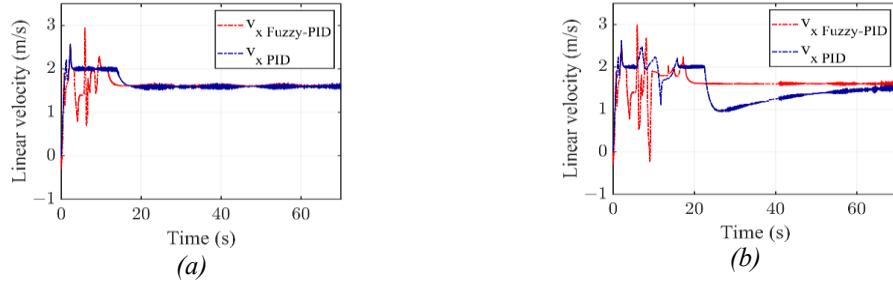


Figure 21. Linear velocity using Fuzzy-PID and PID under (a) zero CoM and (b) displacement CoM condition

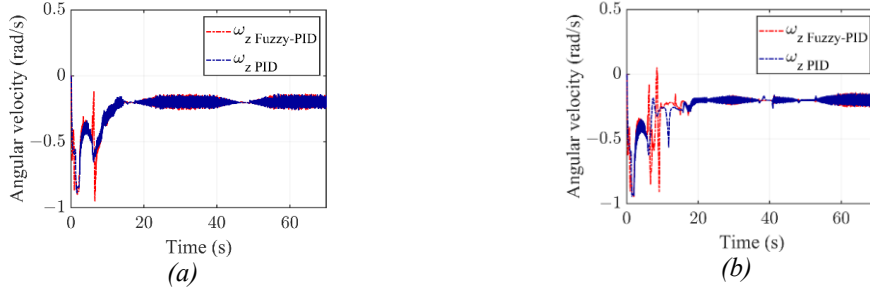


Figure 22. Angular velocity using Fuzzy-PID and PID under (a) zero CoM and (b) displacement CoM condition

5. CONCLUSION

Through the modeling and simulation of the CoM displacement, the loss of control will occur. The more deviation between CoM and geometry center, the larger the unstable occurrences in the motion, especially in the initial stage. This not only affects the wheels' reaction force, but also the linear and angular velocity, which leads to instability. To address this problem, a Fuzzy-PID for both manipulator and platform is designed, which tunes the parameters around the conventional PID's and is compared with it under zero and changing CoM conditions. The simulation of Fuzzy-PID demonstrates a better performance than the conventional PID in platform control, but it still has a poor response in Point-to-Point control of the manipulator. A few gaps remain in modeling and simulation, which only consider the effect of CoM in the x-axis without the y-axis in dynamics modeling. So, in the future, this problem can be addressed with a more complex model, and a controller can be adjusted.

Furthermore, the motor and driver transfer functions vary across different motors, resulting in inconsistent response times that make control more challenging. To address this issue in the future, a coupling for wheels can be proposed, along with a synchronous motor controller to ensure the motion.

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Additional information

Author contributions

Trong Thanh Nguyen served as the main contributor, developed the simulation algorithms, and wrote the the manuscript; Bao Gia Tran formulated the mechanical concepts and developed the case study; Trong Minh Phan Nguyen conducted the motor experiments; Tri Thanh Pham assisted technique and supported the data collection process; Thien Minh Tran supervised the research, managed project administration, and critically reviewed and edited the manuscript. All authors have read and agreed to the published version of the manuscript.

Conflict of interest

The authors confirm that there is no conflict of interest to declare for this publication.

Artificial intelligence (AI) use

The authors accept full responsibility for the content of the manuscript, including the disclosure of any use of AI. During the preparation of this manuscript, the authors used Grammarly to improve the language and grammar, and ChatGPT to improve readability of the text.

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